Logarithmic-Quadratic Slide Rule

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The slide rule has become an indispensable tool in almost all technical fields. But it has been neglected in surveying due to two main causes. First because the ordinary slide rules provide inaccurate results, that is, the multiplications of numbers with 3 and 4 digits, as will be required in calculations of surfaces or polygon points, do not achieve the required accuracy.

Second because they lack of a scale suitable for direct calculation of Pythagorean ratios. The importance of just this second statement is demonstrated by the existence of numerous inventions, of which I know three special slide rules, but none successful yet, as, among other things, they provide results in cumbersome way.

The above two objections are now solved by the following slide rule, first because its scales are 5 times longer than those from ordinary slide rules, with the respective equally greater precision. And second because it includes a special scale where a side of a right-angle triangle is directly found from a single setting of the other two and with enough accuracy, as the sources of error are minimized to the lowest degree.

It is 35 cm long, fits easily in the briefcase and is not bulkier than the ordinary slide. It has three different scales:

1 logarithmic to multiply and the like,
1 quadratic, for the calculation of Pythagorean ratios,
1 sine and cosine scales at the back of the slide, particularly suitable for the calculation of polygonal courses.

All scales have a length of 625 mm and are divided by half. In contrast with the slide rule with scales also divided by half designed by Dr. Frank (1903, Journal of Surveying, p. 401 - 405), the halves of the three divided scales are arranged adjacently, with the advantage that the result is always found in a single base line.

A further innovation is the application of Frank's principle to the quadratic scales, but including a triple numbering (fig. IV) to achieve the required accuracy in practice.

Seifert. Logarithmic-Quadratic Slide Rule
Journal of Surveying 1924

The numbering is such that big numbers apply for hypotenuses between 10 - 25 m or 100 - 250 m, encircled numbers are for 25 - 50 m or 250 - 500 m, and small numbers are for 50 - 100 m or 500 - 1000 m or 0 - 10 m. Thus, each interval in 25 series is 4 times bigger than in 50 series, and 16 times bigger than in the 100 scale.

The sine and cosine scale has also double numbering following the slide rule principle, so that to find the result directly, without intermediate calculation. The slide scales are reversed, mainly to avoid misalignment errors in the most common calculations, the multiplication and finding the hypotenuse, and also to create a table of reciprocal numbers.

The missing values of the sine of 0° - 6°, or the cosine of 84º - 90º, are on the back of the rule, where also a lot of space for other tables and formulas is available, arranged in a small chart.

In addition, four standard map scales are attached in the two bevelled longitudinal sides 1:500, 1:1000, 1:625 and 1:1250. The glass cursor is not framed, for a better overview, like the "Free View" one by Dennert.

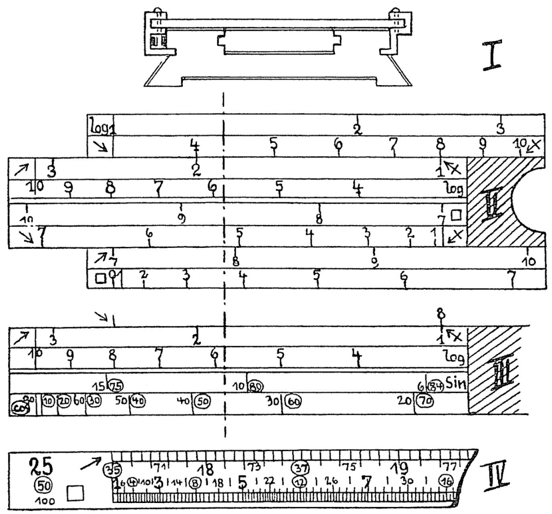

Fig I is a cross-section,
Fig II is a schematic of the whole front of the rule,

Fig III is a schematic of the back of the slide,
Fig IV is a partial view of the real quadratic scales.
The dotted line indicates the cursor line for the following examples, using figures II and III:

A is where the cursor is set in the body scales,
B is the slide number to be placed under the cursor line.

A.		B.		
$\underline{43{,}04}$	·	18,56	=	798,8 ↖
$\sqrt{\underline{36{,}59}^2+\underline{51{,}82}^2}$			=	↗ 63,44
$\overline{136{,}1}$	·	$\overline{\sin 10^\circ 42'}$	=	↘ 25,26
$\overline{13{,}61}$	·	$\underline{\cos 54^\circ 4'}$	=	7,988 ↖

From the examples based in the schematic drawings, the following calculation rule, identical in all three scales, can be seen:

Read product or hypotenuse by ⟶ (left), if initial settings are both in the scales over or under the base lines, and by +→ (right), if the initial settings are one over and one under the base lines (+ sign in the arrow!). Over or under the base line means that the respective number is in the scale half set over or under that base line. Another common rule!

The accuracy of the logarithmic scale results from the following: a 4^{th} digit is directly read from 1 to 2 every five units; from 2 to 5 every 10 units and from 5 to 10 every 20 units. The estimated error is ± 1, ± 2 and ± 3 of the 4^{th} digit.

The degree of precision of the quadratic scale is such that the hypotenuse from 0 to 10 m is read directly to 2 cm (i.e. without estimation), from 10 to 25 m to 5 cm, from 25 to 50 m to 10 cm and from 50 to 100 m to 20 cm. Estimation error as above: it is negligible compared to the inaccuracy of the measurement.

At the angle function scales the sine angle from 5° 30' to 15° can be directly read to 2', from 15° to 30° to 5', from 30° to 50° to 10', from 50° to 70° to 20', from 70° to 80° to 30', from 80° to 85° to 1°, and from 85° to 90° to 5°. *tg* and *ctg* may be found with the slide rule by $\frac{\sin}{\cos}$ or $\frac{\cos}{\sin}$.

To roughly determine the accuracy that can be achieved with the abovementioned slide rule, the average linear estimation error is assumed, when setting or reading, of 0,05 mm. This value may be easily achieved by a precise manufacture of the instrument and with some care in its use. It is assumed from now on that this linear error is the same through all places in the scales. This provides in the logarithmic scale an average total error for the result of a calculation with two arguments, expressed in ‰, of $\frac{0{,}05 \cdot 1000\sqrt{3}}{625 \cdot \text{Mod.}} = 0{,}32‰$.

Reading a m^2	m a m^2	µ a m^2
500	0,16	4,0
1.000	0,32	5,8
2.000	0,64	8,2
4.000	1,28	11,6
6.000	1,92	14,3
8.000	2,56	16,5
10.000	3,20	18,6

From the adjacent table it can be found that the accuracy is sufficient for surface calculations. In column 2 there are the mean errors of the rule results, and shown in column 3 are the mean errors of a surface calculation, derived after the formula $\mu a = \frac{d}{3\sqrt{2}}$ from the maximum error of the cadastral requirements. Major errors will not occur as for factors over 100 m, the product can be easily decomposed.

For calculation of polygonal courses, the accuracy is usually sufficient, especially by the proportional distribution of the calculation errors, that eliminate both the final error and the measurement errors.

In the quadratic scales the error becomes smaller with the reading increase, since the intervals in these scales widen, in contrast to logarithmic scales. Differentiating the equation $x = \frac{a^2 \cdot 625}{z^2}$, where *a* corresponds to the reading length and *z* to the respective final scale number (25, 50 or 100), it results, for a calculation with two arguments: $m_a = \frac{z^2\sqrt{3}}{2a \cdot 625} \cdot m_x$.

With the above assumption of m_x = 0,05 mm the table below can then be calculated. It can be seen that the relative errors listed in column 4 only for one

Seifert. Logarithmic-Quadratic Slide Rule

Journal of
Surveying
1924

value (for 10) are worse than the ones for the logarithmic scales. As a comparison, column 5 is included with the average errors in a length measurement in terrain I, which are derived from the maximum errors of the cadastral requirements, given by the formula $\mu_a = \frac{d}{4\sqrt{2}}$.

Scale z	Reading a m	$m\,a$ cm	$m\,a$ ‰	$\mu\,a$ cm
25	10	0,43	0,43	1,2
	15	0,29	0,19	1,4
	20	0,22	0,11	1,6
	25	0,17	0,07	1,8
50	25	0,69	0,28	1,8
	30	0,58	0,19	2,0
	40	0,43	0,11	2,3
	50	0,35	0,07	2,6
100	50	1,4	0,28	2,6
	60	1,10	0,19	2,8
	80	0,87	0,11	3,1
	100	0,70	0,07	3,7

Here we see again, like in the logarithmic scale, that the average calculation errors account for only a small fraction of the average measurement errors, even in the worst cases.

The fact, due to the specific layout of the quadratic scales, that the mean errors are smaller with the greater of the reading, is also based in having more intermediate divisions towards the end of the scales. Although this means that the error will decrease more slowly than as indicated in the table, the coherence and clarity of the scales are increased.

As with the common slide rules, the expressions like $\frac{a \cdot b}{c}$ can be calculated with the logarithmic scales of this rule without the need to read the intermediate values, and similarly with the quadratic $a^2 + b^2 - c^2$, where the intermediate result $\sqrt{a^2 + b^2 - c^2}$ is squared with the logarithmic scales.

When calculating the height and the base middle points in a triangle with the formula $p = \frac{a^2 + b^2 - c^2}{2a}$ both logarithmic and quadratic scales are used in this way with only three readings for the three parameters sought, getting a proof for accuracy as well.

The advantage of the alternate use of the scales is specially applicable for calculations of small points, coordinate transformations, polygonal courses, etc. But the best service of this new slide rule is that it significantly reduces the field work by the quick and reliable control of right angle triangles.

From the above it appears that this slide rule is particularly suitable for the surveying work, because the laborious and mechanical calculations are done quickly and reliably, and what is more important, with reasonable accuracy. Needless to say, but the aforementioned advantages outstand for any of the practical calculations related with Pythagorean ratios and angular functions.